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The following note by Professor Mac Cullagh, on the attraction of ellipsoids, was read.

"The object of the present note is to show how the final integrations by which the attraction of a homogeneous ellipsoid is found, when the force varies inversely as the square of the distance, may be performed geometrically; and thus to complete the synthetic solution of a celebrated problem. It has been always supposed that the utmost geometry can do is to arrive in a simple way at the differential expressions on which the attraction depends, leaving the further treatment of the question to the integral calculus; but we shall see that, by putting the differential of the attraction under a certain form, the integral is at once obtained, and that in a very elegant shape, by geometry.

"To avoid useless generality, we shall suppose the attracted point to be at the extremity of an axis of the ellipsoid, as it is well known that the solution of this particular case enables us find the attraction wherever the point is placed. Let O be the centre of the ellipsoid, and A, B, C the extremities of its semiaxes, the lengths of which are denoted by a, b, c respectively, a being the greatest, and c the least. And first, suppose the attracted point to be at C, the extremity of the least semiaxis. Let two right lines passing through C, and making respectively the angles ϕ and $\phi + d\phi$ with OC, revolve within the ellipsoid, describing two right cones, of which OC is the common axis, and which include between their surfaces a differential portion dM of the volume of the ellipsoid. The attraction of the matter contained in dM is evidently in the direction of OC.

"Now let us consider the focal ellipse having its centre at O, and lying in the principal plane which is at right angles to OC. Let E be the extremity of the major axis of this ellipse; and putting

$$\rho = \sqrt{c^2 + (b^2 - c^2)\cos^2\phi}, \quad \rho' = \sqrt{c^2 + (a^2 - c^2)\cos^2\phi},$$

take in OE produced a point P such that OP shall be to OE as ρ to c. Then, if a right line drawn from P touch the focal ellipse in the point T, and if the angle OPT be denoted by θ, it will be found that

$$\cos\theta = \frac{\sqrt{a^2 - c^2}}{\rho'} \cos\phi.$$

Suppose that the point P moves to p, when ϕ is changed into $\phi + d\phi$. Then it may be easily shown that the attraction of dM upon the point C is proportional to the interval Pp multiplied by the cosine of θ. In this form the differential of the attraction is immediately integrable. For if from p we draw a right line pt touching the ellipse in t, and if s denote the difference between the tangent PT and the elliptic arc ET, while $s + ds$ denotes the difference between the tangent pt and the arc Et, it will appear, by a lemma which I have frequently had occasion to use (see the Proceedings of the Academy, vol. ii. p. 508), that ds is equal to Pp multiplied by cos θ. The integral, beginning when $\phi = 90°$, or $\rho = c$, is therefore proportional to s. At the other limit we have $\phi = 0$, and $\rho = b$, which determines the extreme position of the point T. The difference between the tangent PT and the elliptic arc ET, corresponding to this position, is to be multiplied by a certain function of the semiaxes, in order to get the whole attraction of the ellipsoid on the point C.

"When the attracted point is at B, the extremity of the mean axis, we proceed exactly as before; but instead of the focal ellipse we make use of the focal hyperbola, whose plane is at right angles to OB. Putting now

$$\rho = \sqrt{b^2 - (b^2 - c^2)\cos^2\phi}, \qquad \rho' = \sqrt{b^2 + (a^2 - b^2)\cos^2\phi},$$

and calling E the extremity of the primary axis of the hyperbola, we take in OE a point P (which will lie between O and E) such that OP shall be to OE as ρ to b. Then, drawing from P the right line PT, touching the hyperbola in T, and denoting the angle EPT by θ, we have

$$\cos\theta = \frac{\sqrt{a^2 - b^2}}{\rho'} \cos\phi;$$

whence it may be shown that if p be the point to which P moves when ϕ becomes $\phi + d\phi$, the interval Pp multiplied by $\cos\theta$ will be proportional to the attraction of the matter dM contained between the surfaces of two right cones having B for their vertex and OB for their common axis, provided ϕ and $\phi + d\phi$ be the angles which the sides of these cones make with OB. The whole attraction is therefore proportional to the difference between the tangent PT and the hyperbolic arc ET, the position of T being that which corresponds to the supposition $\phi = 0$, or $\rho = c$.

" When the attracted point is at the extremity of the greatest axis of the ellipsoid, we cannot employ a similar method, because there is no focal curve perpendicular to that axis. But if A, B, C be the attractions at A, B, C respectively, we have the known relation

$$\frac{\text{A}}{a} + \frac{\text{B}}{b} + \frac{\text{C}}{c} = 4\pi,$$

of which a geometrical proof will be found in the Proceedings, vol. ii. p. 525; and from this relation we can find A in terms of B and C.

" The preceding method of treating the question of the attraction of ellipsoids was given at my lectures in Trinity College, in the beginning of last year. I have since observed that the same results may be obtained, and perhaps more readily, by dividing the ellipsoid into concentric and similar shells. For the attraction of dM is equal, in each case, to the attraction of the shell bounded by the surfaces of two ellipsoids whose semiaxes are $a\cos\phi$, $b\cos\phi$, $c\cos\phi$, and $a\cos(\phi + d\phi)$, $b\cos(\phi + d\phi)$, $c\cos(\phi + d\phi)$, these ellipsoids having O for their centre, and OA, OB, OC for the directions of their semiaxes. And the attraction of such a shell on an external point may be simply expressed by means of the semiaxes of a confocal ellipsoid passing through the point. (See a Memoir by M. Chasles in Liouville's Journal, vol. v.) The quantities which we have called ρ and ρ' are, in fact, semiaxes of an ellipsoid described through the attracted point (that is, through C in the first case, and through B in the second) so as to be confocal to the surface of which the semiaxes are $a\cos\phi$, $b\cos\phi$, $c\cos\phi$."

A note by Professor Mac Cullagh, on the rotation of a solid body, was read.

Let a solid body be made to revolve round a fixed point O, and be afterwards free from any external forces; and through O conceive a right line OI to be drawn perpendicular to the invariable plane (the plane passing through O and the direction of the primitive impulse). If O be the centre of an ellipsoid whose semiaxes are in the directions of the principal axes belonging to that point, and of such lengths that the square of each semiaxis is equal to the corresponding moment of inertia divided by the mass of the body, the motion will take place in such a way that the point I, in which the right line OI intersects the surface of the ellipsoid, will be fixed in space; and therefore OI will describe within the body a cone of the second order, condirective with the ellipsoid (that is, having its circular sections parallel to those of the ellipsoid), while the point I describes on the surface of the ellipsoid a certain spherical conic. In a former number of the Proceedings (vol. ii. p. 542), the author had alluded to a theorem for determining the time at which the point I occupies any given position on the spherical conic, and he now gave a particular statement of it as follows:

Conceiving a plane of circular section of the ellipsoid to be drawn through its mean axis, and the spherical conic to be projected on this plane, first by right lines parallel to the